1 RATIONALS & IRRATIONALS

1. RATIONAL NUMBERS

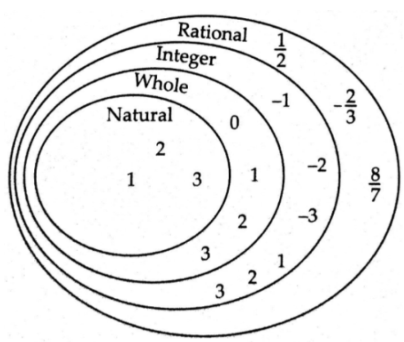
	♦	INTRODUCTION TO RATIONAL NUMBERS
	♦	DECIMAL REPRESENTATION OF RATIONAL NUMBERS

SYNOPSIS-1

The set of real numbers is union of set of rationals (Q) and irrationals (Q')
i.e., R = $Q \cup Q'$

RATIONAL NUMBERS

The number of the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$, is called a rational number.



Ex: $\frac{7}{8}, -\frac{8}{9}, -\frac{5}{9}, \frac{3}{5}$ are all rational numbers.

FRACTIONS AS RATIONAL NUMBERS:

A fraction is in the form $\frac{p}{q}$ where p and q are positive integers and $q \neq 0$.

Therefore, every fraction is a rational number.

INTEGERS AS RATIONAL NUMBERS:

The integer 7 can be written as $\frac{7}{1}$

The integer -7 can be written as $-\frac{7}{1}$.

Therefore, all integers can be written in the form $\frac{p}{q}$ where $q = 1$.

All integers are therefore rational numbers.

DENSITY PROPERTY OF RATIONAL NUMBERS:

Between any two given rational numbers there exists uncountable rational numbers. This property of rational numbers is called the property of density.

Method-1: Let a and b be two rational numbers $q_1 = \frac{a+b}{2} \Rightarrow a < q_1 < b$

q_1 is the rational number between a and b.

$q_2 = \frac{a+q_1}{2} \Rightarrow a < q_2 < q_1 < b$

q_2 is the rational number between a and q_1.

$q_3 = \frac{q_1+b}{2} \Rightarrow a < q_2 < q_1 < q_3 < b$.

q_3 is the rational number between q_1 and b.

In this manner we can find infinite rational numbers between two given distinct rational numbers.

Ex. Give three rational numbers lying between $\frac{1}{3}$ and $\frac{1}{2}$.

Sol. **Method-1:** The rational number lies between $\frac{1}{3}$ and $\frac{1}{2}$ is

$$\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right) = \frac{1}{2}\left(\frac{2}{6}+\frac{3}{6}\right) = \frac{1}{2}\times\frac{5}{6} = \frac{5}{12}$$

$$\Rightarrow \frac{1}{3} < \frac{5}{12} < \frac{1}{2}$$

Second rational number between $\frac{1}{3}$ and $\frac{5}{12}$ is

Also, $\frac{1}{2}\left(\frac{1}{3}+\frac{5}{12}\right) = \frac{1}{2}\left[\frac{4}{12}+\frac{5}{12}\right] = \frac{1}{2}\times\frac{9}{12} = \frac{9}{24} = \frac{3}{8}$

$$\Rightarrow \frac{1}{3} < \frac{3}{8} < \frac{5}{12} < \frac{1}{2}$$

Third rational number between $\frac{5}{12}$ and $\frac{1}{2}$ is

$$\frac{1}{2}\left(\frac{5}{12}+\frac{1}{2}\right) = \frac{1}{2}\left(\frac{5}{12}+\frac{6}{12}\right) = \frac{1}{2}\times\frac{11}{12} = \frac{11}{24}$$

$$\Rightarrow \frac{1}{3} < \frac{3}{8} < \frac{5}{12} < \frac{11}{24} < \frac{1}{2}$$

Hence, $\frac{3}{8},\frac{5}{12},\frac{11}{24}$ are the required three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

Method-2: $\frac{a}{b}$ and $\frac{c}{d}$ be two rational numbers.

Step-1. Make denominators equal in both rational numbers.

Step-2. If we have to find n rational numbers between $\frac{ad}{bd}$ and $\frac{cd}{bd}$, then multiply numerators and denominators by such a number so that the difference between the numerators is at least n.

Method-2: Make the denominator equal by multiplying 2 and 3 $\frac{1}{3}\times\frac{2}{2}=\frac{2}{6}$ and $\frac{1}{2}\times\frac{3}{3}=\frac{3}{6}$

To insert 3 rational numbers, we multiply the numerators and denominator by such a number so that difference between the numerators is at least 3.
Multiplying the numerators and denominators of both fractions by 4, we get $\frac{8}{24}$ and $\frac{12}{24}$.

Hence the required 3 rational numbers are $\frac{9}{12},\frac{10}{12},\frac{11}{12}$.

SYNOPSIS-2

TERMINATING AND NON-TERMINATING DECIMALS

A rational number can be expressed as either a terminating or non – terminating recurring decimal.

Ex: i) $\frac{9}{2}=4.5,\frac{3}{4}=0.75, 2=2.0$ etc., are rational numbers which are terminating decimals

ii) $\frac{4}{3}=1.333......=1.\overline{3},\ \frac{1}{6}=0.1666.......=0.1\overline{6},$

$\frac{9}{11}=0.8181......=0.\overline{81},\ \frac{1}{7}=0.142857142857.....0.\overline{142857},$ etc are non-terminating repeating decimals.

Criterion for rational numbers to be terminating decimals: If a rational number ($\neq$ integer) can be expressed in the form $\frac{p}{2^n\times5^m}$, where p$\in$ Z, n $\in$ W and m $\in$ W then rational number will be terminating decimal. Otherwise, rational number will be non-terminating recurring decimal.

Ex: i) $\frac{3}{8}=\frac{3}{2^3\times 5^0}$ So, $\frac{3}{8}$ is a terminating decimal.

ii) $\frac{7}{250}=\frac{7}{2^3\times 5^3}$ So, $\frac{7}{250}$ is a terminating decimal.

iii) $\frac{8}{75}=\frac{8}{5^2\times 3}$ is a non-terminating recurring decimal.

Conversion of terminating decimals into rational numbers (m/n form where m, n are positive integers)

Method: In the numerator, write the given number leaving out the decimal point. In the denominator, write 1 followed by as many zeros as the number of digits in the decimal part. Reduce the fraction to its simplest form.

Ex: (i) $13.4=\frac{134}{10}=\frac{67}{5}$ (ii) $23.87=\frac{2387}{100}$

(iii) $352.9171=\frac{3529171}{10000}$ (iv) $0.78929=\frac{78929}{100000}$

Conversion of pure recurring decimals into rational numbers (m/n form where m, n are positive integers)

The recurring part of the non - terminating recurring decimal is called period and the number of digits in the recurring part is called periodicity.

Ex: $\frac{1}{3}=0.\bar{3}$ period = 3, periodicity = 1, $\frac{3}{11}=0.\overline{27}$ period = 27, periodicity = 2,

$\frac{5}{13}=0.\overline{384615}$, period = 384615, periodicity = 6

We can express non - terminating recurring decimals in the fraction form of rational numbers:

Ex: Let us find the rational form of $0.\overline{428571}$

The periodicity of the recurring decimal is 6.

So multiply the decimal fraction by 10^6 and subtract the number from the product

$0.\overline{428571}$ = x (say)

$10^6x=1000000x=428571.\overline{428571}$

$(-)x=\quad 0.\overline{428571}$

$999999x=428571$

$\therefore x=\frac{428571}{999999}=\frac{3}{7}$

Conversion of mixed recurring decimals into rational numbers (m/n form where m, n are positive integers)

Ex: Let us write $0.1\overline{25}$ in the form of rational numbers

In $0.1\overline{25}$ periodicity = 2, period = 25

So, multiply by 102 and subtract the number from the product.

Let $0.1\overline{25} = x$

$10^2x = 100x = 12.52525.........$

$(-)\ x = 0.12525.............$

$99x = 12.4$

$$\therefore x = \frac{12.4}{99} = \frac{124}{990} = \frac{62}{495}$$

The method of expressing recurring decimals in the form p/q is as follows.

Recurring decimal = (The whole number obtained by writing the digits in their order)–(The whole number made by the non-recurring digits in order) / 10(The number of digits after the decimal point) – 10(The number of digits after the decimal point that do not recur)

Note: i) $a.b\overline{cd} = \dfrac{abcd - ab}{10^3 - 10^1}$ ii) $a.\overline{bcd} = \dfrac{abcd - a}{10^3 - 10^0}$

Ex: Express $15.0\overline{2}$ as a rational number?

Sol: Here, the whole number obtained by writing digits in their order = 1502

The whole number made by the non-recurring digits in order = 150,

The number of digits after the decimal point = 2 (two)

The number of digits after the decimal point do not recur = (one)

$$\therefore 15.0\overline{2} = \frac{1502 - 150}{10^2 - 10^1} = \frac{1352}{100 - 10} = \frac{1352}{90} = \frac{676}{45}$$

Note: Thus, every rational number can be expressed either as a terminating decimal or non-terminating repeating decimal.

Conversely, we can express terminating and non-terminating but repeating decimals in the rational number form, that is in the form of $\frac{p}{q}$, where p, q ∈ Z and q ≠ 0.

ADDITION, SUBTRACTION, OF RECURRING DECIMALS

ADDITION OF RECURRING DECIMALS

Ex. $5.7\overline{32} + 8.\overline{613}$

Sol: $5.7\overline{32} + 8.\overline{613} = 5 + 0.7\overline{32} + 8 + 0.\overline{613} = 5 + \dfrac{732 - 7}{990} + 8 + \dfrac{613}{999}$

$$=5+\frac{725}{990}+8+\frac{613}{999}=13+\frac{725}{990}+\frac{613}{999}=13+\frac{145}{198}+\frac{613}{999}$$

$$=13+\frac{145\times 3\times 37+613\times 2\times 11}{2\times 3\times 9\times 11\times 37}$$

$$=\frac{13\times 2\times 3\times 9\times 11\times 37+145\times 3\times 37+613\times 2\times 11}{2\times 3\times 9\times 11\times 37}$$

$$=\frac{285714+16095+13486}{21978}=\frac{315295}{21978}=14.3459368$$

Alternatively:

Step 1. Express the numbers without bar as 5.732323232…….. + 8.613613613613………

Step 2. Write the numbers as one above other i.e.,

5.732323232
8.613613613

Step 3. Divide this number into two parts. In the first part i.e., left side write as many digits as there will be integral value with non-recurring decimal. In the right side write as many digits as the LCM of the number of recurring digits in the given decimal number

e.g.,

5.7	323232	(Since 5.7 is the integral + non-recurring part]
8.6	136136	(The LCM of 2 and 3 is 6]

Step 4. Now add or subtract as usual

5.7	323232
8.6	136136
14.3	459368

Sep 5. Put the bar over the digits which are on the right side in the resultant value. $14.3\overline{459368}$

Thus $5.\overline{732}+8.613=14.3\overline{459368}$

SUBTRACTION OF RECURRING DECIMALS

Ex: Solve the following: $19.3\overline{68421}-16.20\overline{53}$

Sol: 19.3684216842168421 – 16.2053535353…

$=19.3684216842168421=16.205353535353535353$

19.36	8421684216
–16.20	5353535353
3.16	3068148863

(Since 16.20 is integral part with non-recurring digits)

[The LCM of 5 and 2 is 10]

In the first no. there are 5 recurring digits and in the second no. there are 2 recurring digits $19.3\overline{68421}-16.20\overline{53}=3.16\overline{3068148863}$

1. RATIONAL NUMBERS

WORK SHEET LEVEL–I MAINS CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

INTRODUCTION TO RATIONAL NUMBERS

1. A number which can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a _______.
   1) Rational number 2) Natural number
   3) whole number 4) Fraction
2. Which of the following statement is False?
   1) Every fraction is a rational number 2) Every rational number is a fraction
   3) Every integer is a rational number 4) All of these
3. Number of rational numbers are there between any two rational numbers?
   1) 0 2) 3 3) 4 4) Infinite
4. A rational number between $\frac{1}{4}$ and $\frac{1}{3}$ is
   1) $\frac{7}{24}$ 2) 0.29 3) $\frac{13}{48}$ 4) All of these
5. The following are any three rational numbers between 3 and 4
   1) $\frac{7}{3},\frac{13}{5},\frac{15}{2}$ 2) $\frac{7}{4},\frac{15}{2},\frac{13}{7}$ 3) $\frac{7}{2},\frac{15}{4},\frac{13}{4}$ 4) None of these

DECIMAL REPRESENTATION OF RATIONAL NUMBERS

6. The recurring part of the non - terminating recurring decimal is called ______
   1) Period 2) Periodicity 3) Both (1) & (2) 4) Neither (1) nor (2)
7. The number of digits in the recurring part of the non - terminating recurring decimal is called
   1) Period 2) Periodicity 3) Both (1) & (2) 4) Neither (1) nor (2)
8. In $0.\overline{35}$ period is
   1) 3 2) 5 3) 35 4) 53
9. In $0.\overline{89}$ periodicity is
   1) 4 2) 6 3) 46 4) 2
10. While converting the rational number into decimal the process of division terminates, then the decimal is ____
    1) Terminating 2) non-terminating
    3) Repeating 4) non-terminating not repeating

LEVEL–II

INTRODUCTION TO RATIONAL NUMBERS

11. Find ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$

1) $\frac{97}{160}<\frac{98}{160}<\frac{99}{160}<\frac{103}{160}<\frac{101}{160}<\frac{102}{160}<\frac{107}{160}<\frac{108}{160}<\frac{110}{160}<\frac{106}{160}$
2) $\frac{97}{160}<\frac{98}{160}<\frac{99}{160}<\frac{100}{160}<\frac{101}{150}<\frac{102}{150}<\frac{103}{150}<\frac{104}{150}<\frac{105}{150}<\frac{106}{160}$
3) $\frac{96}{160}<\frac{98}{160}<\frac{99}{160}<\frac{100}{160}<\frac{110}{160}<\frac{111}{160}<\frac{112}{160}<\frac{114}{160}<\frac{115}{160}<\frac{116}{160}$
4) $\frac{97}{160}<\frac{98}{160}<\frac{99}{160}<\frac{100}{160}<\frac{101}{160}<\frac{102}{160}<\frac{103}{160}<\frac{104}{160}<\frac{105}{160}<\frac{106}{160}$

12. Find any five rational numbers lying between $\frac{2}{7}$ and $\frac{2}{5}$

1) $\frac{21}{70},\frac{22}{70},\frac{23}{70},\frac{25}{70},\frac{26}{70}$ 2) $\frac{19}{70},\frac{20}{70},\frac{22}{70},\frac{24}{70},\frac{26}{70}$
3) $\frac{18}{70},\frac{23}{70},\frac{25}{70},\frac{27}{70},\frac{29}{70}$ 4) $\frac{22}{70},\frac{23}{70},\frac{24}{70},\frac{25}{70},\frac{26}{70}$

DECIMAL REPRESENTATION OF RATIONAL NUMBERS

13. The $\frac{m}{n}$ form of $0.\overline{437}$ is

1) $\frac{437}{999}$ 2) $\frac{437}{900}$ 3) $\frac{437}{990}$ 4) $\frac{437}{909}$

14. $0.\overline{2}+0.\overline{3}+0.\overline{4}+0.\overline{9}+0.\overline{39}$

1) $0.\overline{57}$ 2) $1\frac{20}{33}$ 3) $2\frac{1}{3}$ 4) $2\frac{13}{33}$

LEVEL-III ADVANCED CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

15. The fraction form of $12.\overline{148}$ is

1) $\frac{4009}{333}$ 2) $\frac{4019}{333}$ 3) $\frac{1436}{990}$ 4) $\frac{12136}{999}$

16. Find the value of $2.\overline{6}-0.\overline{9}$

1) $\frac{5}{3}$ 2) $\frac{1}{3}$ 3) $\frac{7}{2}$ 4) $\frac{1}{2}$

17. Simplify $0.88\overline{5}-0.3\overline{53}$

1) $0.63\overline{10}$ 2) $0.43\overline{30}$ 3) $0.53\overline{10}$ 4) $0.53\overline{20}$

LEVEL–IV

STATEMENT TYPE QUESTIONS

18. Statement I: One of the rational between $\frac{1}{5}$ and $\frac{1}{4}$ is $\frac{9}{40}$

Statement II: If x and y are any two rational numbers such that x < y, then $\frac{1}{2}(x+y)$ is a rational number between x and y such that $x < \frac{1}{2}(x+y) < y$

1) Both Statements are true.
2) Both Statements are false.
3) Statement I is true, Statement II is false.
4) Statement I is false, Statement II is true.

19. Statement I: $\frac{1}{0}$ is rational number

Statement II: $\frac{p}{q}$ is rational number, if q $\neq$ 0

1) Both statements are true
2) Both statements are false
3) Statement I is true, Statement II is false
4) Statement I is false, statement II is true

INTEGER TYPE QUESTIONS

20. The length of the period of the decimal form of $\frac{289}{13}$ is __________6

21. $\frac{1}{2}$ can't be expressed as rational number having denominator 5 because 5 is not the multiple of _______

MULTI CORRECT ANSWER TYPE QUESTIONS

22. A non-terminating decimal form the following are

1) $\frac{3}{11}$ 2) $\frac{81}{64}$ 3) $\frac{9}{100}$ 4) $\frac{11}{17}$

23. Which of these are not reciprocals of rational numbers lying between 1 and 4?

1) $\frac{3}{4}$ 2) $\frac{4}{3}$ 3) $\frac{5}{17}$ 4) $\frac{2}{9}$

LEVEL–V

COMPREHENSION TYPE QUESTIONS

PASSAGE:

From a starting point A, Rahul walks $\frac{3}{4}km$ towards east and then $\frac{6}{7}km$ towards west to reach point C.

24. Where will he be now from the starting point A?

1) $\frac{9}{28}$ towards west
2) $\frac{3}{28}$ towards west
3) $\frac{3}{28}$ towards east
4) $\frac{9}{28}$ towards east

25. How much total distance Rahul walks to reach point C?

1) $\frac{45}{28}km$
2) $\frac{43}{28}km$
3) $\frac{47}{28}km$
4) $\frac{49}{28}km$

26. How much more distance he walked towards east than west?

1) $\frac{-9}{28}$
2) $\frac{3}{28}$
3) $\frac{9}{28}$
4) $\frac{-3}{28}$

MATRIX MATCH TYPE QUESTIONS

27. Match the following

	Column-I		Column-II
a	$\frac{201}{202}$ expressed as a	p	Terminating decimal
b	$\frac{201}{256}$ expressed as a	q	$\frac{402}{458}$
c	The number which is lies between $\frac{201}{202}$ and $\frac{201}{256}$ are	r	Repeating decimal
d	$\left(\frac{p}{q}\times\frac{m}{n}\right)\times\frac{x}{y}=\frac{p}{q}\left(\frac{m}{n}\times\frac{x}{y}\right)$ is a non-repeating	s	Non-terminating
		t	Associative law of multiplication

2. IRRATIONAL NUMBERS

♦	INTRODUCTION TO IRRATIONAL NUMBERS
♦	PROPERTIES OF IRRATIONAL NUMBERS

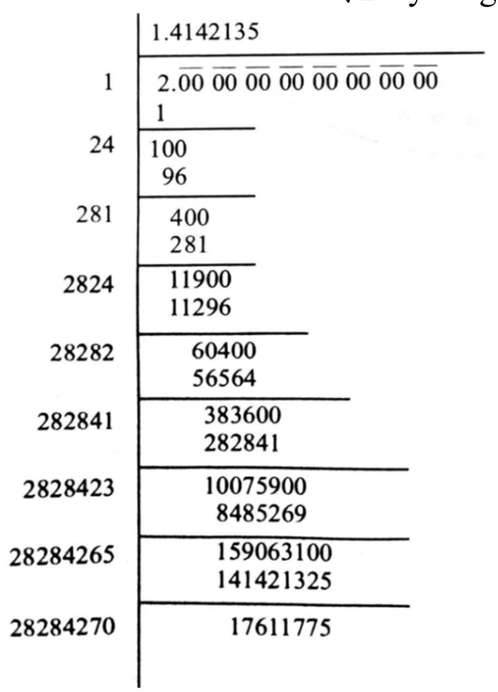
SYNOPSIS-1

SQUARE ROOF OF A POSITIVE RATIONAL NUMBER:

Let 'a' be any positive rational number and we express $\sqrt{a} = b$ if and only if b> 0 and b^2 = a, the value of b is called the 'positive square root' of positive rational number 'a'.

Ex. Find the value of $\sqrt{2}$ upto five decimal places

Sol: Let us find the value of $\sqrt{2}$ by long division method



Step 1 : After 2, place decimal points.

Step 2 : After decimal points write 0's.

Step 3 : Group '0' is pairs and put a bar over them.

Step 4 : Then after follow the method of find the square root of perfect square.

Note : i) The value of $\sqrt{2}$ up to five decimal places is 1.41421.

ii) The value of $\sqrt{3}$ up to five decimal places is 1.7 3205.

IRRATIONAL NUMBERS: Some decimal fractions are neither terminating nor recurring decimals.

Ex. i) 3.112123123412345……. ii) 1.732050807…….

In the first example, there appears to be a pattern are structure. It is neither terminating nor recurring decimal. The second one is also neither terminating nor recurring decimal. But there seems to be no pattern or structure. The decimal fractions cannot be expressed in the form of rational numbers, and therefore they are called irrational numbers.

DEFINITION: A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number.
Thus, non-terminating, non-repeating decimals are irrational numbers. These are denoted by 'S' or 'Q'.

Examples of Irrational Numbers:

Type 1: i) Clearly, 0.01001000100001... is a non-terminating and non-repeating decimal and therefore, it is irrational,
ii) 0.12112111211112..., 0.54554555455554... are irrationals.

Type 2: The 5th Century BC the Pythagorean in Greece, the follower of the famous mathematician and philosopher Pythagorean, proved that $\sqrt{2}$ =1.4142135623731...is an irrational number. Later Theodoras of Cyrene showed that $\sqrt{3},\sqrt{5},\sqrt{6},\sqrt{10},\sqrt{11},\sqrt{12},\sqrt{13},\sqrt{14},\sqrt{15}$ and $\sqrt{17}$ are also irrational numbers.

Type 3: The number 'e' (Euler's number) is an irrational. Its value is 2.71828182845...i.e., 2 < e <3

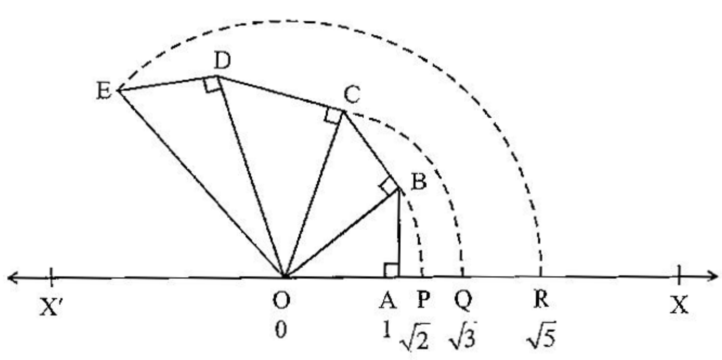
Know about π :

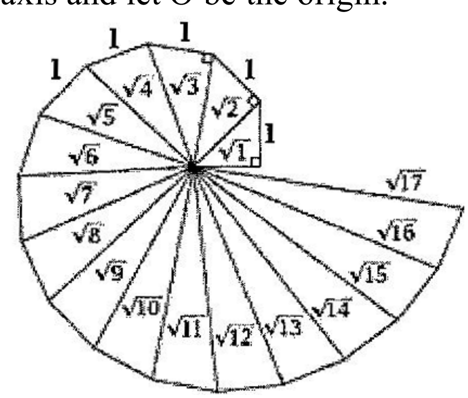
π= 3.141592653589793238 . . .
The decimal expansion of % is non-terminating non-recurring. So π is an irrational number. Note that, we often take $\frac{22}{7}$ as an approximate value of π, but $\pi \neq \frac{22}{7}$. We celebrate March 14th as π day, since it is 3.14 and time 1:59 (as π = 3.14159) what a coincidence, Albert Einstein was born on March 14th, 1879.

REPRESENTING IRRATIONAL NUMBERS ON NUMBER LINE:

Ex. Represent each of the numbers $\sqrt{2},\sqrt{3}$ and $\sqrt{5}$ on the real line.

Sol: Let X'OX be a horizontal line, taken as the X-axis and let O be the origin.





Let O represents 0.
i) Take OA= 1 unit and draw $AB \perp OA$ such that AB = 1 unit.

Join OB. Then, $OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$ units

Write O as centre and OB as radius, drawn on arc, meeting OX at P.

Then, OP = OB = $\sqrt{2}$ units

Thus, the point P represents $\sqrt{2}$ on the real line.

ii) Now, draw $BC \perp OB$ such that BC = 1 unit.

Join OC. Then, $OC = \sqrt{OB^2 + BC^2} = \sqrt{\left(\sqrt{2}\right)^2 + 1^2} = \sqrt{3}$ units

With O as centre and OC as radius, draw an arc, meeting OX at Q, then OQ = OC = $\sqrt{3}$ units.

Thus, the point Q represents $\sqrt{3}$ on the real line.

iii) Now. draw $CD \perp OC$ such that CD = 1 unit.

Join OD. Then, $OD = \sqrt{OC^2 + CD^2} = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = \sqrt{4} = 2$ units

Now, draw $DE \perp OD$ such that DE = 1 unit

Join OE. Then, $OE = \sqrt{OD^2 + DE^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$ units _

With O as centre and OE as radius draw an arc, meeting OX at R.

Then, $OR = OE = \sqrt{5}$ units.

Thus, the point R represents $\sqrt{5}$ on the real line.

Hence, the points P, Q, R represent the numbers $\sqrt{2}, \sqrt{3}$ and $\sqrt{5}$ respectively.

Property: If a and b are two positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between a and b.

Ex. Find three irrationals between 3 and 4.

Sol: An irrational number between 3 and 4 is $\sqrt{12}$

An irrational number between 3 and $\sqrt{12}$ is $\sqrt{3\sqrt{12}} = \sqrt{6\sqrt{3}}$

Another irrational number between $\sqrt{12}$ and 4 is $\sqrt{4\sqrt{12}} = \sqrt{8\sqrt{3}}$

$\therefore$ Required numbers are $\sqrt{6\sqrt{3}}, \sqrt{12}$ and $\sqrt{8\sqrt{3}}$ lie between 3 and 4.

Ex: Find two irrational numbers between 2 and 3.

Sol: An irrational number between 2 and 3 is $\sqrt{2 \times 3} = \sqrt{6}$.

Similarly, an irrational number between 2 and $\sqrt{6} = \sqrt{2 \times \sqrt{6}} = \sqrt{2\sqrt{6}}$

$\therefore$ Required numbers are $\sqrt{6}$ and $\sqrt{2\sqrt{6}}$ lie between 2 and 3.

PROPERTIES OF IRRATIONAL NUMBERS:

1. Irrational numbers satisfy the commutative, associative and distributive laws for addition and multiplication.

2. i) Sum of two irrationals need not be an irrational.

Ex. Each one of $\left(2+\sqrt{3}\right)$ and $\left(4-\sqrt{3}\right)$ is irrational.

But, $\left(2+\sqrt{3}\right)+\left(4-\sqrt{3}\right)=6$, which is rational,

ii) Difference of two irrationals need not be an irrational.

Ex. Each one of $\left(5+\sqrt{2}\right)$ and $\left(3+\sqrt{2}\right)$is irrational.

But $\left(5+\sqrt{2}\right)-\left(3+\sqrt{2}\right)=2$, which is rational,

iii) Product of two irrationals need not be an irrational.

Ex. $\sqrt{3}$ is irrational. But $\sqrt{3}\times\sqrt{3}=3$, which is rational,

iv) Quotient of two irrationals need not be an irrational.

Ex. Each one of $2\sqrt{3}$ and $\sqrt{3}$ is irrational. But $\frac{2\sqrt{3}}{\sqrt{3}}=2$, which is rational.

2. IRRATIONAL NUMBERS

WORK SHEET LEVEL–I MAINS CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

INTRODUCTION TO IRRATIONAL NUMBERS

1. Numbers which cannot be expressed in the form $\frac{p}{q}, q \neq 0, p, q \in Z$ are called _____
   1) rational 2) irrational 3) fractional 4) decimal
2. $\sqrt{2}$ is an _______ number
   1) rational 2) irrational 3) natural 4) fractional
3. Decimal representation of $\sqrt{2}$ to first four places is _____
   1) 1.4145 2) 1.4143 3) 1.4142 4) 1.4144
4. The number of irrationals in the given list $\sqrt{3}, \pi, \frac{1}{3}, 0, \sqrt[5]{2}, \frac{22}{7}, \sqrt{36}$ is
   1) 3 2) 4 3) 5 4) 6
5. Number of irrational numbers between any two rational numbers is
   1) 1 2) 2
   3) 10 4) Infinite

PROPERTIES OF IRRATIONAL NUMBERS

6. The sum of two Irrational numbers is
   1) Either Rational or Irrational number
   2) Always Rational numbers
   3) Always Irrational numbers 4) All of the above
7. Which among the following is not true?
   1) Difference of two Irrational number is either Rational or Irrational
   2) Product of two Irrational number is either Rational or Irrational
   3) Quotient of two Irrational number is always Irrational
   4) Sum of two Rational number is always Rational
8. Identify the incorrect property of Irrational number?
   1) An Irrational number can be approximated as closely as we like by Irrational numbers
   2) There are Infinitely many Irrational numbers between two rationals
   3) There are finite number of Irrationals between any two numbers
   4) The set of Irrational numbers dense every where
9. Identify the correct property of Irrational numbers?
   1) Product of two Irrationals is Irrational
   2) Division of one Rational and one Irrational is either Rational or Irrational
   3) Quotient of one Rational and are Irrational is either Rational or Irrational
   4) Difference of a Rational and an Irrational is always Irrational

LEVEL–II

INTRODUCTION TO IRRATIONAL NUMBERS

10. The two irrational numbers between $\sqrt{6}$ and $\sqrt{8}$

1) $\sqrt{7}$, $\sqrt{\frac{15}{2}}$ 2) $\sqrt{11}$, $\sqrt{13}$ 3) $\sqrt{12}, \sqrt{17}$ 4) $\sqrt{19}, \sqrt{21}$

11. The two rational numbers between $\sqrt{5}$ and $\sqrt{7}$

1) 5.6, 1.2 2) 4.5, 3.1 3) 2.4, 2.5 4) 5.7, 7.8

PROPERTIES OF IRRATIONAL NUMBERS

12. An irrational number from the following

1) $\left(2+\sqrt{3}\right)+\left(4-\sqrt{3}\right)$ 2) $\left(2+\sqrt{3}\right)^2$

3) $\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)^2$ 4) $\left(2+\sqrt{2}\right)\left(2-\sqrt{2}\right)$

13. Irrational among the following is

1) $\sqrt{2}\times\sqrt{2}$ 2) $\sqrt{21}\times\sqrt{\frac{3}{7}}$ 3) $\sqrt{2\times3\times5\times7}$ 4) $\sqrt{2\times\frac{2008}{1004}\times1}$

14. If $\sqrt{8} > 2.82$ then value of $\frac{2\sqrt{8}}{\sqrt{2}}$ is equal to

1) 6 2) 8 3) 4 4) 2

LEVEL-III ADVANCED CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

15. If $\sqrt{5}=2.236$ and $\sqrt{10}=3.162$, then the value of

$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$ is

1) 5.398 2) 4.398 3) 3.398 4) 6.398

16. The value of $\sqrt{7\sqrt{7\sqrt{7\sqrt{7}}}}$ is

1) 0 2) 7

3) $7^{\frac{15}{16}}$ 4) $\sqrt{7}$

LEVEL–IV

STATEMENT TYPE QUESTIONS

17. Statement I: Product of one Rational number and one irrational number is always Irrational (Except when Rational number $\neq 0$)
Statement II: $\sqrt{10}\times 10 = 24.62$
1) Both the statements are true
2) Both the statements are false
3) Statement I is true, statement II is false
4) Statement I is false, statement II is true.

18. Statement I: $3\sqrt{3}+5\sqrt{3}-\sqrt{27}=5\sqrt{3}$
Statement I: $a\sqrt{d}+b\sqrt{d}-c\sqrt{d}=(a+b-c)\sqrt{d}$
1) Both the statements are true
2) Both the statements are false
3) Statement I is true, statement II is false
4) Statement I is false, statement II is true.

INTEGER TYPE QUESTIONS

19. The value of $\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)=$ ____________

20. If $a=2+\sqrt{3}+\sqrt{5}$ and $b=3+\sqrt{3}-\sqrt{5}$, then $a^2+b^2-4a-6b-3=$ ___

MULTI CORRECT ANSWER TYPE QUESTIONS

21. Which of the following is not Irrational?
1) π 2) e
3) $\frac{22}{7}$ 4) 3.12304200179112

22. The two irrational numbers between $\sqrt{6}$ and $\sqrt{8}$
1) $\sqrt{7}$ 2) $\sqrt{\frac{15}{2}}$ 3) $\sqrt{11}$ 4) $\sqrt{13}$

23. The rational numbers between two irrationals $\sqrt{17}$ and $\sqrt{37}$ are
1) 5 2) 6 3) 7 4) 8

LEVEL–V

COMPREHENSION TYPE QUESTIONS

PASSAGE:

Find the value of the variables in the following and identify whether it is Rational or Irrational.

24. If $a = \left(7 - \sqrt{7}\right)^2$, then

1) $a = 14 - 56\sqrt{7}$ and it is Irrational
2) $a = 56 + 14\sqrt{7}$ and it is Irrational
3) $a = 56 - 14\sqrt{7}$ and it is Irrational
4) $a = 14 + 56\sqrt{7}$ and it is Irrational

25. If $b = \left(\sqrt{5} + \frac{1}{\sqrt{5}}\right)^2$, then

1) b = 1 and it is Rational
2) $10 - \frac{2}{\sqrt{5}}$ and it is Irrational
3) $\frac{36}{5}$ and it is Rational
4) $36 - 2\frac{1}{\sqrt{5}}$ and it is Irrational

26. $\left(10 + \sqrt{5}\right)\left(10 - \sqrt{5}\right)$ is

1) 20 is Rational
2) $100 - \sqrt{15}$ is Irrational
3) 95 is Rational
4) $5 + \sqrt{15}$ is Irrational

MATRIX MATCH TYPE QUESTIONS

27.

COLUMN – I	**COLUMN – II**
a) Product of two irrationals	p) Rationals
b) Sum of rational and irrational	q) Irrational
c) Difference of two irrationals	r) May be or may not be irrationals
d) Quotient of two irrationals	s) Itself